Fresnel lens solar concentrator derivations and simulations

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ABSTRACT

Fresnel lens solar concentrators continue to fulfill a market requirement as a system component in high volume cost effective Concentrating Photovoltaic (CPV) electricity generation. Design and optimization may be performed using comprehensive system simulation tools, but before investing in the effort to build a complete virtual model framework, much insight can be gathered beforehand by generating a parameterized simulation cache and referencing those results.

To investigate the performance space of the Fresnel lens, a fast simulation method which is a hybrid between raytracing and analytical computation is employed to generate a cache of simulation data. This data is post-processed to yield results that are not readily achieved via derivation. Example plots that can be used for look-up purposes will be included.

Lens parameters that will be interrogated include focal length, index of refraction, prism fidelity, aperture, transmission and concentration ratio. In order to compactly represent a large variety of lens configurations, some variables that define the Fresnel lens will be parameterized.

Analysis will be limited to Fresnel lens prisms oriented toward the photovoltaic (PV) cell and the plano surface directed toward the sun. The reverse of this configuration is rarely encountered in solar concentration applications and is omitted.

Keywords: fresnel lens, polymer optics, micro-optics, parameterized lens design, illumination, transmission efficiency, lens transmittance, prism fidelity

1. INTRODUCTION

A wealth of background information on Fresnel lenses is available in the literature. See for instance Aničin et al.1 for a historical overview and Leutz and Suzuki2 for a text specific to Fresnel lenses for solar concentration. This investigation will be primarily concerned with flat Fresnel lenses in a form factor suitable for PV solar concentrators.

The sample data presented is limited to Acrylic (PMMA) based lenses. With the proper material characteristic information, a similar analysis can be carried out for Silicone-on-Glass (SOG) based technology and this is likely a high value study to be completed in the future.

Typically, solar concentrator environmental durability requirements mandate that the prismatic surface of the Fresnel lens be oriented away from the sun and toward the internal PV cell (herein referred to as Grooves-Facing-Short-Conjugate or GFSC configuration – see Fig. 1).

In order to apply the results of this investigation to a wide range of possible lens geometries, plot data is reported in terms of the ratio of the lens focal length to the aperture size. This ratio is commonly referred to as the f-number or focal ratio (also written as $f/#$) and is defined as $f/# = f/CA$ where $f$ is the effective-focal-length of the lens, and CA is the lens optical clear aperture diameter. In the solar concentrator GFSC arrangement the effective-focal-length and back-focal-length are essentially equivalent.

The definition of the clear aperture (CA) is unambiguous for a circular lens (as shown in Fig. 1: “Circular Fresnel Lens Aperture”), but for a square lens, clear aperture may be defined by the diameter that circumscribes the square, inscribes the square or somewhere in between. This investigation will adopt the convention of defining the lens clear aperture used to compute the $f/#$ of a square lens as the circumscribed diameter (that is the clear

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Circular Fresnel Lens Aperture
Square Fresnel Lens Aperture

Fig. 1. Typical Grooves-Facing-Short-Conjugate Fresnel lens solar concentrator embodiment illustrating both circular and square aperture shapes.

Aperture diameter is defined as the diagonal distance between two lens corners – see Fig. 1: “Square Fresnel Lens Aperture”). The caveat of this being that when comparing results between circular and square lenses, a circular lens with the same $f/#$ as a square lens defined per the circumscribed $f/#$ has a factor of $\pi/2$ more active area.

The prism pitch refers to the projected width of individual concentric prism rings. Varied schemes can be enlisted to define the pitch values over the Fresnel lens face (for instance constant pitch vs. constant depth). For the most part though, the idealized lens throughput is independent of the pitch scheme and can be adjusted to maximize manufacturing quality of the lens. Consequently, the analysis will not be dependent on the Fresnel lens prism pitch. Further, prism pitch can be used to parameterize out pitch dependent loss factors such as draft and prism tip peak rounding. With reference to Fig. 2, the rounding ratio ($R$) is defined as the quotient of the peak radius ($r$) and the prism pitch ($d$):

$$R = \frac{r}{d}$$  \hspace{1cm} (1)

Similarly, the draft fraction ($D$) is the projected draft ($b$) over the prism pitch ($d$):

$$D = \frac{b}{d}$$  \hspace{1cm} (2)

2. METHODOLOGY

When considering an analytical approach to performance analysis of the optical properties of a Fresnel lens solar concentrator it is most straightforward to take advantage of the circular symmetry found in the multitude of concentric rings that constitute the refractive power of the lens. As more detailed optical characteristics are considered, (such as prism peak tip radius defects), a direct analytical model becomes more burdensome and a hybrid raytracing approach may be implemented as in Davis.\textsuperscript{3} However this approach still exploits circular symmetry and is only correct for circular aperture Fresnel lenses. In order to properly simulate square based Fresnel lenses (which are much more commonly applied in solar concentrators) a new approach is developed. In addition, accuracy is improved by including wavelength dependence, the AM1.5 spectral irradiance distribution, material absorbance and a variable prism tip radii model generated from manufacturing experiments as discussed in Davis et al.\textsuperscript{4}

For any point on the surface of the Fresnel lens, the following parameters may be computed:

$T_{\text{surf}}$ The transmittance accounting for surface reflections at the plano input and prism slope surfaces.
Fig. 2. Geometric model for Fresnel lens prism peak rounding.

$T_{\text{geom}}$ The transmittance accounting for the prism draft surface loss and the loss due to the prism peak tip rounding.

$T_{\text{matl}}$ The transmittance accounting for the material absorbance as light is transmitted through the lens media.

Note that it is assumed there is no aperture dependence (any additional loss due to propagation through prism facets is negligible).

Then, the spectrally weighted transmission efficiency or transmittance of a Fresnel lens can be reasonably defined as:

$$T_{\text{eff}} = \frac{\int_{\lambda} T_{\text{matl}} \int_{A} T_{\text{surf}} \cdot T_{\text{geom}} \cdot dA \cdot d\lambda}{\int_{\lambda} \int_{A} dA \cdot d\lambda} \tag{3}$$

where $A$ represents lens surface area and $\lambda$ is wavelength.

It is a feasible task to solve Eq. 3 for a Fresnel lens with circular grooves centered within a circular aperture by integrating the differential area ($dA$) in polar coordinates. Since there are terms in the integrand that are not readily expressed analytically, in application, the integrals become summations and the sample size is selected sufficiently small so that the summations approximate the integrals. Throughout this report, equations will be presented in integral form, but they are practically implemented as summations on finite increments.

For the circular case, the terms in the area integrand are polar symmetric, so integrating over a circular aperture in polar coordinates is natural. For a square aperture however, the differential area needs to be summed over rectangular coordinates. This requires application of a proper change of variables for the analytic and discrete terms in the integrand. The complexity of this approach is eschewed in favor of the following:

A set of variables which define a specific Fresnel lens geometry is selected (aperture shape, aperture size, focal length, material optical properties and material thickness). An array of spatial coordinates ($x, y$) are generated to be uniformly distributed over the aperture at some selected sample resolution (see Fig. 3). Each spatial coordinate approximates a square differential area bin on the lens. Within any bin, all of the component transmittance terms in Eq. 3 can be calculated. At a sufficiently small sample bin size, summation of these terms yields a good approximation to the lens transmittance. The lens aperture size is then incremented, the lens area is resampled and the computation is repeated. In this way, a cache of data is generated over a large combination of aperture sizes with a fixed focal length. The data is easily parameterized to $f/#$ by ratioing the aperture size to the focal length.

Computation can be done quickly in data language software (for instance NumPy as used in this study) and it is still much leaner than a conventional raytrace allowing for exploration over a large geometrical space and relatively fine grain increments over the solar spectral wavelength regime.
This analysis assumes a Fresnel lens design type which forms a point focus at a select design wavelength. However the Fresnel lens primary of a concentrator system may be a nonimaging type of optic designed without regard for point focus characteristics. The results produced herein should be considered of utility for Fresnel lens primaries where the optical path deviates only modestly from a point focus imaging type Fresnel lens. It is postulated that this will be valid for a wide variety of designs and has utility even for nonimaging Fresnel lenses.

In addition, a method for approximating the maximum beam size at the focal plane is implemented. This is useful for computing the concentration ratio.

2.1 Material Index of Refraction

For an accurate simulation, real index of refraction data vs. wavelength is desirable. This was acquired experimentally by molding a plano-plano witness sample of the same acrylic polymer material used in Fresnel lens solar concentrator fabrication. The index of refraction was characterized using a Metricon prism coupler. Data is plotted in Fig. 4.

2.2 Coordinate Radialization

In order to properly compute ray vector transmission through 3D geometric entities, 2D simplified refraction calculations in a single plane will not suffice. However, due to the natural circular symmetry of the Fresnel lens, the normal vectors can be reduced to 2D by changing the coordinate reference plane to a radial section. This is useful for simplifying calculations that have inherent polar symmetry (e.g. draft and radius loss). At any coordinate location on the lens surface \((x, y)\), the prism normal can be expressed as \(\vec{N} = (N_x, N_y, N_z)\). In polar coordinates, \((x, y)\) can be expressed as a radial distance: \(\rho = \sqrt{x^2 + y^2}\). Then for any \(\rho\) coordinate, the normal vector is \(\vec{N}_r = (N_r, N_z)\) where \(N_r = \sqrt{N_x^2 + N_y^2}\) will be called the radial normal.

The radial normal defines the radial slope \((N'_r)\):

\[
N'_r = \frac{N_r}{N_z}
\] (4)

At any Fresnel lens coordinate \((x, y)\) the Cartesian coordinate normal vector \(\vec{N}\) can be expressed as:

\[
(N_x, N_y, N_z) = \left(\frac{N_x x}{\sqrt{(1 + N_x^2)(x^2 + y^2)}}, \frac{N_y y}{\sqrt{(1 + N_y^2)(x^2 + y^2)}}, \frac{1}{\sqrt{(1 + N_z^2)}}\right)
\] (5)

Radial ray vectors can be expressed as \(\vec{U}_r = (u_r, u_z)\) before entering the lens, \(\vec{U}'_r = (u'_r, u'_z)\) inside the lens and \(\vec{U}''_r = (u''_r, u''_z)\) after exiting the prisms.
2.3 Transmittance adjusted for surface reflection ($T_{\text{surf}}$)

The following equation from Davis:\(^3\)

$$N_{\text{GFSC}} = \frac{u''_y}{\sqrt{1 - u''^2_y}} - n_i$$  \hspace{1cm} (6)

Can be rewritten as:

$$N_r = \frac{1}{f/\rho - n_i \sqrt{1 + (f/\rho)^2}}$$  \hspace{1cm} (7)

Where at the design wavelength, the material index of refraction is $n_i$ and the lens back-focal-length is $f$. Then the radial slope $N_{\text{GFSC}} = N_r/N_z$ is found for the $\rho$ value at each coordinate bin (it is also useful to recognize the radial slope angle from the plane of the optic is: $\theta_r = \tan^{-1} N_r$).

Once the normal vector is known for any coordinate location, the Fresnel surface reflection and transmission formulae can be applied to account for the surface transmittance through the plano and prismatic interfaces of the Fresnel lens. These formulae are detailed in various optics texts; for example Born and Wolf\(^6\) and Hecht and Zajac\(^7\) are often cited. They can then be used for each coordinate bin to yield the $T_{\text{surf}}$ transmittance:

$$t_\perp = \frac{2 \sin \theta_i \cos \theta_i}{\sin(\theta_i + \theta_t)}$$
$$t_\parallel = \frac{t_\perp}{\cos(\theta_i - \theta_t)}$$

$$T_{\text{surf}} = \left( \frac{n_t \cos \theta_t}{2n_i \cos \theta_i} \right) \left( t_\perp^2 + t_\parallel^2 \right)$$  \hspace{1cm} (8)

Where $n_i$ is the index of refraction for the ray in the incident media, $n_t$ is the index of refraction in the transmitted media, and $\theta_i$ and $\theta_t$ are the incident and refracted angles respectively in the plane of incidence.

2.4 Transmittance adjusted for geometry loss ($T_{\text{geom}}$)

The geometry loss arises from the draft surface and prism tip rounding of the Fresnel lens facets. In the GFSC concentrator configuration, assuming predominately collimated light input, a model for the geometric loss is
derived in Davis with a small error. The correct equation should be:

\[ T_{\text{geom}} = 1 - \frac{s + b}{d} \]  \hspace{1cm} (9)

where \( s \) is the radius loss distance and \( b \) is the projected draft (see Fig. 2). Applying the ratios from Eqs. 1 and 2, this can be rewritten as:

\[ T_{\text{geom}} = 1 - R \tan \left( \frac{\pi/2 + \theta - \phi}{2} \right) \cos \theta - \mathcal{D} \]  \hspace{1cm} (10)

with \( \theta \) representing the radial slope angle with respect to the plane of the optic and \( \phi \) being the radial draft angle with respect to the normal of the optic (see Fig. 2).

At this point, another useful parameterization to define is the aspect ratio (\( A \)) as the quotient between the prism height (\( h \)) and the prism pitch (\( d \)):

\[ A = \frac{h}{d} = \frac{\tan \theta}{1 + \tan \theta \tan \phi} \]  \hspace{1cm} (11)

Then the draft fraction (\( \mathcal{D} \)) of Eq. 2, can be expressed as:

\[ \mathcal{D} = A \tan \phi \]  \hspace{1cm} (12)

Assuming a radial draft angle value of approximately \( \phi = 3\degree \) over the full aperture, the draft fraction can be approximated as \( \mathcal{D} \approx A/20 \).

Next the rounding ratio (\( \mathcal{R} \)) might be considered a fixed value over the lens aperture, however previous work shows a functional dependence of the rounding ratio on the aspect ratio (see Fig 5). The rounding ratio can be expressed as \( \mathcal{R} = \mathcal{R}(A) \) where \( \mathcal{R}(A) \) is some known function based on experimental results from the manufacturing process conditions being utilized. Now Eq. 10 can be expressed completely as a function of the radial slope angles and radial draft angles:

\[ T_{\text{geom}} = 1 - \mathcal{R}(A(\theta, \phi)) \tan \left( \frac{\pi/2 + \theta - \phi}{2} \right) \cos \theta - A(\theta, \phi) \tan \phi \]  \hspace{1cm} (13)

So at any coordinate bin, the geometric transmittance \( T_{\text{geom}} \) can be computed.

### 2.5 Transmittance adjusted for material absorbance (\( T_{\text{mat}} \))

The \( T_{\text{mat}} \) term in Eq. 3 compensates for the material absorbance. The absorption of radiation through a homogeneous media is governed by the Beer-Lambert Law:

\[ T_{\text{int}} = e^{-\alpha t_0} \]  \hspace{1cm} (14)

where \( T_{\text{int}} \) is the internal transmission through a path length thickness of \( t_0 \) in a media with an absorption coefficient of \( \alpha \).

One method of measuring \( T_{\text{int}} \) is to subject a plano-plano witness sample of the optical media to a spectrophotometric scan. If the index of refraction (\( n \)) vs. wavelength profile of the sample is well known, then the theoretical surface reflectance terms can be removed from the measured data leaving just \( T_{\text{int}} \). Consider a plano-plano sample oriented normally with respect to a substantially collimated sample beam and a spectrophotometric scan which yields transmittance values of \( T_t \), then:

\[ T_{\text{int}} = T_t \cdot \frac{(n + 1)^4}{16n^2} \]  \hspace{1cm} (15)

Accurately knowing the sample thickness (\( t_0 \)), Eq. 14 can be used to solve for the absorption coefficient (\( \alpha \)). But since it is the internal transmission at an objective thickness we seek, we can calculate this directly; assuming a
known internal transmission ($T_{\text{int}}$) for a measured sample thickness ($t_0$), the internal material transmission ($T_{\text{matl}}$) through an objective thickness ($t$) is:

$$T_{\text{matl}} = (T_{\text{int}})^{t/t_0}$$  \hspace{1cm} (16)

For this study, a material objective thickness of $t = 3\,\text{mm}$ was implemented. The internal transmission ($T_{\text{int}}$) for a plano-plano witness sample thickness of $t_0 = 2.7\,\text{mm}$ was spectrophotometrically characterized and is plotted in Fig. 6. Using Eq. 16 then $T_{\text{matl}}$ for any wavelength can be found. This transmission is uniformly applied to all spatial coordinates since it is not aperture dependent.

### 2.6 Solar Spectral Irradiance Weighting

With solar concentration under interest, it will be of great utility to include the solar spectral irradiance as a weighting factor when computing the Fresnel lens transmittance. Let $I_0(\lambda)$ equal the direct plus circumsolar spectral irradiance for Air Mass 1.5 (see Fig. 7). Then Eq. 3 can be modified for the Fresnel lens solar spectral weighted transmission efficiency ($T_{\text{solar}}$):

$$T_{\text{solar}} = \frac{\int_\lambda I_0(\lambda)T_{\text{matl}}\int_A T_{\text{surf}} \cdot T_{\text{geom}} \cdot dA d\lambda}{\int_\lambda I_0(\lambda) \int_A dA d\lambda}$$  \hspace{1cm} (17)

Additional scale factors may be applied in this simulation process based on application specific assembly knowledge (for example, the PV cell spectral response). However this investigation does not do any additional scaling beyond spectral irradiance weighting.

### 2.7 Spot Size

Up to this point, the methodology presented yields the Fresnel lens solar spectral weighted transmission efficiency ($T_{\text{solar}}$) for an arbitrary $f/#$ lens of circular, square or even arbitrary aperture shape. The values computed should be of reasonable accuracy assuming substantially collimated light input.

For computing the optical concentration ratio however, the beam area at focus is required and assuming a perfectly collimated source model will not give results indicative of performance. The direct solar disk has a...
Fig. 6. Measured Acrylic transmission through 2.7mm of material thickness with theoretical surface reflectance contributions removed.

Fig. 7. Spectral Irradiance Reference for Air Mass 1.5 Direct plus Circumsolar.

half-angular extent of 4.65 mrad (± 0.2664°), and to allow for alignment tolerances an acceptance cone of ± 2° may also be considered. This report will evaluate a uniform field for these two angular extents. Further work may consider the full direct plus circumsolar sunshape distribution.

A good statistical representation of the spot size at the focal plane requires a large sampling of rays over the acceptance cone of the Fresnel lens. However, since we only have a single collimated ray per spatial coordinate bin on the lens, we seek a way to estimate the spot size. Using a radialized coordinate system as per Sec. 2.2, the angular deviation of a ray through the lens may be calculated via Snell’s Law:
\[
\begin{align*}
  u''_r &= n \left( \frac{u_r - N_r u'_r}{n} \right) + N_r \sqrt{1 + N_r^2 - n^2 \left( \frac{u_r - N_r u'_r}{n} \right)^2} \\
  &\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \Quad
Nevertheless we will use the metric with the postulate that it establishes an upper bound on the spot diameter and consequently a lower bound on the concentration ratio.

Next, the transmission efficiency per coordinate value \( T_{\text{coord}} \) is defined as:

\[
T_{\text{coord}} = T_{\text{mat}} \cdot T_{\text{surf}} \cdot T_{\text{geom}} \tag{25}
\]

Each \( T_{\text{coord}} \) associated with a new spot radius value \( \rho' \) is analogous to a flux vector given a unity flux input distribution. We can use these values to compute a wavelength dependent flux weighted RMS spot radius in the focal plane \( \rho'_{\text{rms}}(\lambda) \). Assuming \( M \) spatial coordinate bins over the lens aperture:

\[
\rho'_{\text{rms}}(\lambda) = \sqrt{\frac{\sum_{i=1}^{M} (\rho'_{\text{rms}})_{\lambda i} \cdot (T_{\text{coord}})_{\lambda i} \cdot \sum_{i=1}^{M} (T_{\text{coord}})_{\lambda i}}{M}} \tag{26}
\]

Then the solar spectral weighted RMS spot radius \( \rho_{\text{solar}} \) is defined as:

\[
\rho_{\text{solar}} = \frac{\int_{\lambda} I_{0}(\lambda) \cdot \rho'_{\text{rms}}(\lambda) \, d\lambda}{\int_{\lambda} I_{0}(\lambda) \, d\lambda} \tag{27}
\]

To make the results generally applicable to a wide range of lens geometries, it will be helpful to parameterize the spot size on the clear aperture. In a similar manner as focal ratio or \( f/# \) is defined, we will define the spot ratio or \( S/# \) as:

\[
\frac{S}{#} = \frac{2\rho_{\text{solar}}}{CA} \tag{28}
\]

The RMS spot diameter is immediately available then by multiplying the spot ratio \( (S/#) \) with the lens clear aperture \( (CA) \).

### 2.8 Concentration Ratio

The spectral weighted optical concentration ratio \( C(\lambda) \) will be defined as:

\[
C(\lambda) = \frac{\eta(\lambda)A}{\pi \rho^2_{\text{solar}}(\lambda)} \tag{29}
\]

Where \( A \) is the Fresnel lens area and \( \eta(\lambda) \) is the spot efficiency for the flux contained within the RMS spot radius \( \rho^2_{\text{solar}}(\lambda) \). Knowing the flux distribution as a function of radial distance in the focal plane \( F(\rho') \) from an incident flux distribution of \( F_{0}(A) \), the spot efficiency \( \eta(\lambda) \) employed in Eq. 29 is:

\[
\eta(\lambda) = \frac{\int_{0}^{\rho'_{\text{solar}}(\lambda)} F(\rho') \, d\rho'}{\int_{A} F_{0}(A) \, dA} \tag{30}
\]

And the integrated solar concentration ratio \( C_{\text{solar}} \) is defined to be:

\[
C_{\text{solar}} = \frac{\int_{\lambda} C(\lambda) \cdot I_{0}(\lambda) \, d\lambda}{\int_{\lambda} I_{0}(\lambda) \, d\lambda} \tag{31}
\]
3. RESULTS

The program is run for both circular and square apertures for 3mm thick Acrylic. The aperture is sampled using spatial coordinate bins of 0.5mm. A fixed focal length of 100mm is selected. Wavelength values are stepped at 50nm increments over the 300nm to 2500nm solar spectrum. The design wavelength is chosen to be 500nm. A fixed radial draft angle of 3° is assumed.

Upon completion, the program writes out a large cache of data which may be post-processed for analytical purposes. In the following sections, selected data sets are culled from the cache and examined in detail.

3.1 Component Transmittances

At the design wavelength, the component transmittances $T_{surf}$ (Sec. 2.3), $T_{geom}$ (Sec. 2.4) and $T_{matl}$ (Sec. 2.5) are plotted in Fig. 9. In addition data acquired from raytracing the same geometry definition using commercial nonsequential raytracing software yields excellent agreement with the $T_{surf}$, $T_{geom}$ product. Inclusion of the material absorption is plotted as the product $T_{surf} \cdot T_{geom} \cdot T_{matl}$ which is equivalent to the transmission efficiency per coordinate ($T_{coord}$).

![Fig. 9. Plotted Transmittance components at the design wavelength for a square Fresnel lens concentrator. Also data is included for a nonsequential raytrace indicating very good agreement with the model.](image)

3.2 Spectral Weighted Transmittance

As per Sec. 2.6, $T_{solar}$ is computed by integrating over the solar spectrum shown in Fig. 7. This is performed for circular and square aperture lenses at different rounding conditions and plotted in Fig. 10.

Comparison of the various peak rounding conditions plotted in Fig. 10 indicates that a high quality manufacturing process (“Condition-1, High Precision Molding”) yields results practically equivalent to the idealized “No Rounding” condition. However, depending on the $f/#$, about 1-2% loss is incurred using the “Condition-3, typical injection molding” process. Data presented from this point forward need only include “No Rounding” and “Condition-3” rounding since “Condition-1” deviation from ideal is negligible.

From Fig. 10, it appears that for a circular aperture lens it will be beneficial to keep the $f/#$ greater than $f/0.7$ and for a square aperture lens, greater than $f/0.6$. At these values the solar transmittance is approximately
74% and decreasing the f/# further causes the efficiency to drop sharply. If the f/# is kept above f/1.3 for circular and f/1.2 for square aperture lenses, better than 80% transmittance may be realized. Continuing to increase the f/# from here only asymptotically improves efficiency up to around 82%.

As discussed in Sec. 1, the f/# for the square lens is defined using the circumscribed diameter of a circular clear aperture (Fig. 1: “Square Fresnel Lens Aperture”). For this reason, there is limited meaning in a one-for-one comparison using f/# between the transmission efficiency of a circular lens with a square lens. Indeed, referencing Fig. 10, it appears a square lens will have better transmittance at a given f/# compared with it’s circular counterpart. To make a fair comparison, a metric should be used which parameterizes the lens power on the available lens area (instead of clear aperture). To accomplish this, we define the quantity squared focal ratio as \( f^2/A \) where \( A \) is the active optical lens area; a circular aperture lens will have \( A = \pi (CA/2)^2 \) and a square aperture lens will have \( A = CA^2/2 \). Plotting transmission efficiency vs. \( f^2/A \) as in Fig. 11 provides an unbiased data space for comparing differently shaped lens apertures. Per the figure, a circular lens has marginally better transmittance vs. a square lens of equivalent squared focal ratio. Of course a square lens has advantageous array packing when compared with a circular lens, but this is not herein under investigation.

### 3.3 Spot Size

Using the method detailed in Sec. 2.7, the solar spectral weighted RMS spot radius \( \rho_{\text{solar}} \) is estimated and used for defining the spot ratio \( (S/#) \) from Eq. 28. The results are plotted in Fig. 12.

As noted in Sec. 2.7, Eq. 24 produces calculated spot sizes that are much larger than RMS spot sizes computed from a statistical raytrace (~2-4x). Since it is understood that this may be the case because of the undue weighting applied to the edge of the field caused by \( \rho'_{\text{new}} \) (Eq. 24) when calculating \( \rho'_{\text{new}}(\lambda) \) (Eq. 26), the data should be interpreted as an upper limit for the spot size. But it is important to realize that even as a limiting function, it is not necessarily a constant offset from actual. Further work is necessary to investigate this offset and to improve the model.

Given the aforementioned caveat on the accuracy of the represented spot ratios, Fig. 12 shows the substantial difference between the ±0.266° direct solar subtense and the ±2° acceptance cone fields. The spot ratio increases for higher f/# lenses (particularly noticeable for the ±2° field). As the f/# is decreased the spot ratio decreases and then inflects (for the ±2° field circular aperture lens, the inflection is around f/0.9 and for the square aperture lens around f/0.7). After the inflection, the spot ratio increases rapidly. This is likely due to the increased transverse chromatic aberration at the high angles of refraction associated with low f/#’s.

![Fig. 10. Solar Spectral Weighted Transmission Efficiency (T_solar) plots for Circular and Square Aperture Lenses.](image-url)
Circular vs. Square Lens Aperture Transmittance
plotted against Squared Focal Ratio $f^2/A$.

Fig. 11. Circular vs. Square Lens Aperture Transmittance comparison when plotted on an axis of Squared Focal Ratio $f^2/A$.

Solar RMS Spot Ratio for CIRCULAR LENS APERTURE

Solar RMS Spot Ratio for SQUARE LENS APERTURE

Fig. 12. Solar RMS Spot Ratio ($S/#$) for Circular and Square Lens Apertures. RMS Spot Diameter and Focal Length are available by multiplying $S/#$ and $f/#$ by the Lens Clear Aperture (CA) respectively.

3.4 Concentration Ratio

From Sec. 2.8, the integrated optical solar concentration ratio ($C_{solar}$) is computed for square and circular apertures with and without prism peak rounding at the $\pm 0.266^\circ$ direct solar subtense and $\pm 2^\circ$ tolerant acceptance cone (see Fig. 13). Due to the large scale difference between the $\pm 0.266^\circ$ and the $\pm 2^\circ$ configurations, they are plotted independently with circular and square aperture data overlayed.

Since the concentration ratio calculation is directly dependent on the spot radius, the results must be considered carefully as previously discussed. Spot radii that are unduly large will yield concentration ratios that are too small. Keeping this in mind, referencing Fig. 13, a circular aperture lens achieves peak concentration around
Fig. 13. Integrated Solar Concentration Ratio ($C_{\text{solar}}$) for the ±0.266° direct solar subtense and a ±2° tolerant acceptance cone.

$f/0.95$ for the ±0.266° field and $f/0.85$ for the ±2° field. A square aperture lens achieves peak concentration around $f/0.7$ for the ±0.266° field and $f/0.8$ for the ±2° field. The peak concentration for the ±0.266° field is approximately 700x and for the ±2° field about 36x. The reduction in concentration ratio due to “Condition-3” prism peak rounding is most evident in the $f/0.6-f/1.5$ regime.

Recall that this analysis assumes an imaging based point focus Fresnel lens. Concentration ratio may be considerably adjusted by other factors. For example, inclusion of a secondary optical element.

4. CONCLUSIONS

A computational method which is a hybrid between analytic formulae and raytracing was discussed and implemented. The method generates a comprehensive cache of data allowing for rapid and insightful analysis. The method has a speed advantage compared to conventional statistical raytracing due to efficient data language based matrix computations and no requirement to generate representative surface geometry or make ray-intercept predictions. It is particularly helpful for dynamically creating a model simulation for geometric loss factors such as prism tip peak rounding and draft angle.

Various optical properties for flat acrylic polymer Fresnel lens solar concentrator primary lenses were investigated over a landscape of possible geometries. Further it was found that transmission efficiency results were in excellent agreement with a statistical raytrace simulation thus validating the approach. However, the lack of statistical ray data required the derivation of an approximation for the RMS spot radius which yielded enlarged spot radii compared to raytracing.

Data was presented for both circular and square aperture Fresnel lenses and although the data plots seem to indicate noticeably better performance for square based lens apertures, this was an artifact of defining $f/#$ for the square aperture based on a circumscribed clear aperture. A proper comparison between aperture types was proposed by defining the squared focal ratio which showed similar performance between circular apertures and square apertures (with circular apertures performing marginally better – see Fig. 11).

Further work may include: running simulations for lens prescriptions generated at different design wavelengths; examination of different polymer materials, particularly Silicone-on-Glass (SOG); modification of prism draft angles; exploration for an improved spot diameter model; inclusion of multi-junction PV cell sensitivity; implementing a direct plus circumsolar sun source angular distribution model; and generation of cell irradiance maps and through-focus plots.
REFERENCES


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